https://doi.org/10.15407/ufm.21.02.249

M.O. KURIN *
National Aerospace University 'Kharkiv Aviation Institute',
17 Chkalov Str., UA-61070, Kharkiv, Ukraine

* m.kurin@khai.edu

DETERMINATION OF THE BOUNDARIES OF PLASTIC ZONE OF METAL DEFORMATION DURING THE CUTTING

The main objective of this work is to analyse the problem of determining the boundary of elastoplastic zone with various methods of machining parts by cutting. The structure of complex theoretical and experimental studies of energy–power parameters of the technological processes is considered. The method for calculating the processes of plastic deformation of metals based on a closed set of equations of continuum mechanics is proposed for the theoretical study of energy–power parameters of the technological processes. The expressions, which make possible the reproduction of the spatial pattern of the strain distribution within the metal at the diamond smoothing and grinding, are obtained. This allows visualizing the mechanism of the deformation and simplifying the analysis of the deformed state of the material. Functional relationship between the power of the deformation and parameters of the machining conditions at the diamond smoothing and grinding is established. Various methods for determining the cutting forces during machining with chip removal as well as approaches to determining deflected mode of a material are considered. A method for express calculation of cutting forces using well-known engineering techniques is proposed. The experimental and calculated data on determination of the sizes of plastically deformable zone of difficult-to-cut materials are analysed. The mechanism of inhibition of dislocations and energy conversion during deformation is considered in detail. As a result, a dislocation–kinetic approach is developed, based on the concept of dislocation as a quasi-particle of a strain quantum. Using the dislocation–kinetic approach, the mathematical model is developed, which allows us to calculate a magnitude of the zone of leading cold hardening that is confirmed by comparison with experimental data. The Starkov's model is improved; the physical meaning of coefficient in formulas for calculating boundaries of cold-hardening zones is explained. A new similarity criterion is introduced, which relates dissipation of plastic strain energy and rate of rearranging of temperature field.

Keywords: elastoplastic zone, cutting forces, dislocation–kinetic approach, similarity criterion, dissipation of energy.

Citation: M.O. Kurin, Determination of the Boundaries of Plastic Zone of Metal Deformation During the Cutting, *Progress in Physics of Metals*, **21**, No. 2: 249–273 (2020)

1. Introduction

The determination of dimensions of elastoplastic zone of a deformed metal during mechanical processing and after its completion is of great practical interest both for predicting changes in the physical-mechanical properties of the surface layer of workpiece and for constructing an integrated picture of the plastic deformation of a metal flowing around a cutting wedge or abrasive grain. In our recently published article [1], we pointed out that the key and open issue of the theory developed based on the hyperbole method is the mechanism for determining the coordinates of boundaries region the onset of plastic metal flow.

The paper [2] has indicated that principal characteristic of localized plastic-flow development known as the elastoplastic invariant of deformation. It is investigated for several different metals. As shown, the distribution of the experimentally obtained values of the invariant can be described by the normal distribution low. As established by the authors, the principal characteristics of autowave processes of localized plasticity development, for instance, the rate and dispersion, can be calculated. It is also possible to calculate the relations between the scales of localized plasticity development as well as the dependence of autowave length on the structure characteristics of metals. In another review [3], the development of techniques for determining the plasticity of materials by the indentation is considered with an introduction of a new plasticity characteristic. This new plasticity characteristic is easily determined by standard determination of hardness by the diamond pyramidal indenters at constant load; thus, the indentation have been proposed for a simple method for determination of the complex of mechanical properties of materials in a wide temperature range using a sample in the form of a metallographic specimen.

The investigation of strain hardening was also in the scope of attention of the authors in Ref. [4], where the authors considered the strain hardening of the metal surface during jet abrasive machining. The functional dependences depth of hardened layer and cold-hardening degree on the technological parameters of jet abrasive are studied.

The determination of the boundaries of the elastic-plastic zone is going to finish the construction of a complete and decidable theory of the process of metal deformation during cutting, covering all processes and related phenomena of the subject area under consideration. The theory development taking into account the above requirements will be aimed at solving problems related to formation of the surface layer during various types of machining, calculating the energy–power characteristics of various processing methods, the kinematics of metal flow during deformation, the formation of a dislocation substructure, *etc.*

2. Analysis of the Deformed State of Material of a Part

2.1. Structure of Investigation of Energy–Power Parameters of Machining Processes

It is necessary to develop mathematical models that reflect relationship of the functional characteristics process with technological parameters of processing modes to predict effectively the energy–power characteristics of various types of machining by pressure, rolling, or cutting. The correct construction of a model is possible to be provided if a structural logical scheme is developed, which defines methods and sequence of theoretical and experimental researchers. The most suitable for calculating processes of plastic deformation are methods based on a closed system equations of continuum mechanics [5–7]. In this case, the deformable metal is considered as an idealized continuous medium with averaged mechanical properties of a real metal.

A theoretical analysis of the majority of technological processes along with the conducted experiments allows us to determine the nature of dependence velocity of particles of plastically deformable metal on coordinates. The velocity of particles can be represented through a velocity vector (cf. with Ref. [5]):

$$\mathbf{V} = \upsilon_x\mathbf{i} + \upsilon_y\mathbf{j} + \upsilon_z\mathbf{k}. \tag{1}$$

The law constancy of a volume during the deformation is expressed by the continuity equation [5]:

$$\mathrm{div}\mathbf{V} = 0. \tag{2}$$

Using Eqs. (1) and (2), we can determine the form of functional dependence of speed on coordinates. Thus, the particles velocity field of material is determined, which makes it possible to calculate the strain rates and their intensity using the formulas:

$$\begin{aligned}\varepsilon_{q_1q_1} &= \frac{1}{H_1}\frac{\partial V_{q_1}}{\partial q_1} + \frac{V_{q_2}}{H_1H_2}\frac{\partial H_1}{\partial q_2} + \frac{V_{q_3}}{H_1H_3}\frac{\partial H_1}{\partial q_3},\\ \varepsilon_{q_1q_2} &= \frac{1}{H_2}\frac{\partial V_{q_1}}{\partial q_2} + \frac{1}{H_1}\frac{\partial V_{q_2}}{\partial q_1} - \frac{V_{q_1}}{H_1H_2}\frac{\partial H_1}{\partial q_1} - \frac{V_{q_2}}{H_1H_2}\frac{\partial H_2}{\partial q_1},\end{aligned} \tag{3}$$

where q_1, q_2, and q_3 are orthogonal curvilinear coordinates.

In this case, the coupling equations hold:

$$x = x(q_1, q_2, q_3),\ y = y(q_1, q_2, q_3),\ z = z(q_1, q_2, q_3);$$

$$H_k = \sqrt{\sum_{i=1}^{3}\left(\frac{\partial x_i}{\partial q_k}\right)^2}; \tag{4}$$

$$\varepsilon_i = \frac{\sqrt{3}}{2}\sqrt{\left(\varepsilon_{q_1q_2} - \varepsilon_{q_2q_2}\right)^2 + \left(\varepsilon_{q_2q_2} - \varepsilon_{q_3q_3}\right)^2 + \left(\varepsilon_{q_3q_3} - \varepsilon_{q_1q_1}\right)^2 + \frac{3}{2}\left(\varepsilon^2_{q_1q_2} + \varepsilon^2_{q_2q_3} + \varepsilon^2_{q_3q_1}\right)^2};$$

here, H_k are Lamé parameters.

Then, it is necessary to determine components of deformation to find the energy–power process parameters,

$$\varepsilon_{11} = \int \varepsilon_{11} dt, \quad \varepsilon_{22} = \int \varepsilon_{22} dt,$$

and deformation intensity,

$$e_i = \frac{\sqrt{3}}{2}\sqrt{\left(e_{11} - e_{22}\right)^2 + \left(e_{22} - e_{33}\right)^2 + \left(e_{33} - e_{11}\right)^2 + \frac{3}{2}\left(e_{12}^2 + e_{23}^2 + e_{31}^2\right)}.$$

An important characteristic of the machining processes is work of deformation, which allows us to determine the power parameters. The total work of deformation is determined by integrating elementary work over volume v:

$$A = \int_t \iiint_v \left(\rho \mathbf{V} \frac{\partial \mathbf{V}}{\partial t} + \mathbf{P}_x \frac{\partial \mathbf{V}}{\partial x} + \mathbf{P}_y \frac{\partial \mathbf{V}}{\partial y} + \mathbf{P}_z \frac{\partial \mathbf{V}}{\partial z} \right) dv dt \,. \tag{5}$$

Two functions were introduced in Ref. [5]. The first is called a velocity function:

$$L = \rho \mathbf{V} \frac{\partial \mathbf{V}}{\partial t} \,. \tag{6}$$

Part of Eq. (5) expresses the energy dissipation function:

$$E = \mathbf{P}_x \frac{\partial \mathbf{V}}{\partial x} + \mathbf{P}_y \frac{\partial \mathbf{V}}{\partial y} + \mathbf{P}_z \frac{\partial \mathbf{V}}{\partial z} \,. \tag{7}$$

The speed function is a work aimed at increasing kinetic energy of elementary volume of metal in deformation process. The energy dissipation function is that part of the work contributes to an own deformation of the material. Taking into account Eqs. (6) and (7), we rewrite Eq. (5) in the following form:

$$A = \int_t \iiint_v \left(L + E \right) dv dt. \tag{8}$$

If deformation is carried out at a low speed, then, velocity function has a sufficiently small value in comparison with the energy dissipation function and can be neglected. In this case, the work of deformation will be determined through the function of energy dissipation:

$$A = \int_t \iiint_v E dv dt. \tag{9}$$

The work of deformation for elementary volume metal assigned to octahedral sites has the following form:

$$dA = 3\tau_{\mathrm{oct}} \gamma_{\mathrm{oct}} dv dt, \tag{10}$$

where τ_{oct} is a shear stress on the octahedral site, *i.e.* platform, inclined to the main axes; γ_{oct} is an octahedral strain rate. The octahedral stress relates to the stress intensity as

$$3\tau_{\text{oct}} = \sqrt{2}\sigma_i. \quad (11)$$

The equation relating octahedral strain rate to the strain rate intensity has form

$$\gamma_{\text{oct}} = \frac{1}{\sqrt{2}}\varepsilon_i. \quad (12)$$

Substituting Eqs. (11) and (12) into Eq. (10) and integrating the latter over volume and time, we obtain:

$$A = \int_t \iiint_v \sigma_i \varepsilon_i dv dt. \quad (13)$$

Comparing Eqs. (9) and (13), we can find that

$$E = \sigma_i \varepsilon_i. \quad (14)$$

Thus, it is possible to determine the field of stresses, strains, and energy–power parameters of process using the basic laws of plastic deformation and the equations of continuum mechanics if we know the particles velocity components of deformable metal during mechanical processing.

2.2. Theoretical Study of the Deformed State of a Part Material during the Diamond Smoothing

Theoretical investigation of the deformed state of a part material during diamond smoothing was carried out in a cylindrical coordinate system under the assumption of plastic contact conditions between a hard spherical indenter and deformable half-space. The basics results of theoretical researchers are presented in Refs. [7–9]. We were able to obtain the field of the metal flow velocity in deformation zone based on the general equations of continuum mechanics, using structural logic diagram presented in Ref. [6] for analysing deformed state of part material during the smoothing [9, 10]:

$$V_z = \frac{AR}{t_k}\sin\left(A\left(1-\frac{t}{t_k}\right)\right)\frac{St_k}{\pi r}\left(1-\frac{z}{kSt_k}\right)^2 \sin\frac{\pi r}{St_k};$$
$$V_r = \frac{2ARS}{\pi^2 kr}\sin\left(A\left(\frac{t}{t_k}-1\right)\right)\left(\frac{z}{Skt_k}-1\right)\left(1-\cos\frac{\pi r}{St_k}\right); \quad (15)$$
$$V_\theta = 0.$$

here, R is the indenter radius; t and t_k are the current time and the time of deformation of the treated surface area, respectfully; $A = \text{arctg}(\sqrt{2RH - H^2}/(R - H))$; H is indenter penetration depth; S is the longitudinal feed rate; r and z are coordinates of a point in a cylindrical coordinate system; k is the coefficient of proportionality.

The velocity field was described based on the following assumptions:

(i) the deformation propagation depth is linearly related to the radius of contact zone $h \approx kl$, where k is coefficient of proportionality, l is size of indenter penetration zone in the radial direction;

(ii) the propagation zone of deformation in the radial direction is determined by the equality $b = 2l$;

(iii) the tangential component of velocity is zero taking into account axial symmetry and absence of twisting and drops ($V_\theta = 0$);

(iv) the vertical component of velocity (V_z) can be represented as a product of two functions, each of which is a function of only one argument: $V_z = f(r)\varphi(z)$, where $f(r)$ is function designating the law of change motion metal particles along the r coordinate, which is determined by shape tool, zone distribution of deformations, and shape of the scallop around zone of deformation.

This dependence according to [11] can be represented as:

$$f(r) = V_0 \frac{l}{\pi r} \sin \frac{\pi r}{l}, \tag{16}$$

where V_0 is an indenter speed, r is a point coordinate. Since in practice, traditionally with diamond smoothing, the depth of penetration of a spherical tip $H < 0.3R$, we can use for calculations the equality $l \approx \sqrt{2RH}$, where $H = f(t)$. In our case, $l = St_k$, where S is the longitudinal feed rate.

Function $\varphi(z)$ determines the attenuation law by the z coordinate. According to Ref. [12], this function can be written as

$$\varphi(z) = \left(1 - \frac{z}{kSt_k}\right)^2. \tag{17}$$

We will find velocity of material points along the axis z:

$$V_0 = \frac{\partial z}{\partial t} = \frac{AR}{t_k} \sin\left(A\left(1 - \frac{t}{t_k}\right)\right). \tag{18}$$

Taking into account all the above, the dependence (16) takes the following form:

$$f(r) = \frac{AR}{t_k} \sin\left(A\left(1 - \frac{t}{t_k}\right)\right) \frac{St_k}{\pi r} \sin \frac{\pi r}{St_k}. \tag{19}$$

After substituting Eqs. (17) and (19) into formula for V_z, we finally obtain expression for the vertical component of velocity:

$$V_z = \frac{AR}{t_k} \sin\left(A\left(1 - \frac{t}{t_k}\right)\right)\left(1 - \frac{z}{kSt_k}\right)^2 \frac{St_k}{\pi r} \sin \frac{\pi r}{St_k}. \tag{20}$$

The component of velocity field of metal flow in radial direction V_r (formula (15)) was determined from the continuity equation (conditions of constant volume).

2.3. Investigation of Power–Strength Parameters of the Grinding Process

The study of the energy–power characteristics of the grinding process was carried out based on the hypothesis of a generalized cutter with a continuous cutting edge. Part of the abrasive grains acting over the bond identified with rotational ellipsoids. The essence of the proposed methodology is based on the conceptual model of abrasive grain that it is the equivalent that reflects all the cutting properties of the abrasive wheel. Establishing a direct correlation makes it possible to find the value of the forward angle of the equivalent grain γ, which is quantitatively related to the coefficient of the reference curve:

$$\mathrm{tg}|\gamma| = \frac{1}{\nu}. \qquad (21)$$

The cutting properties of the abrasive tool do not depend on the values of the grinding mode parameters, and γ is meant to be a constant that quantitatively expresses the cutting properties of a particular brand of abrasive tool.

Theoretical research has resulted in analytical dependences, not only describing the metal flow in the deformation zone, but also being suitable for calculating the energy–power characteristics of the process, without the use of a large array of empirical dependences that do not fully reflect the physics of the process and are limited by the narrow scope of experimental research.

Based on the velocity field, which are given by parametric equations (22) [1, 13]

$$V_x(x,y) = V_0\left[\omega(x,y)\right]^{-\frac{1}{2}}\left[\frac{(x\cos\alpha - y\sin\alpha)e^2\sin\alpha + y}{\sqrt{e^2-1}}\right],$$

$$V_y(x,y) = V_0[\omega(x,y)]^{-\frac{1}{2}}\left[\frac{(x\cos\alpha - y\sin\alpha)e^2\cos\alpha - x}{\sqrt{e^2-1}}\right],$$

$$\omega(x,y) = \frac{e^2\left[x_0\cos\alpha \pm \sqrt{\sin^2\alpha(1-e^2)([a(x,y)]^2(e^2\sin^2\alpha - 1) - x_0)}\right]^2}{(1-e^2\sin^2\alpha)^2} + $$
$$+[a(x,y)]^2,$$

$$a(x,y) = \sqrt{\frac{(x\cos\alpha - y\sin\alpha)^2 e^2 - (x^2+y^2)}{(1-e^2)}}, \qquad (22)$$

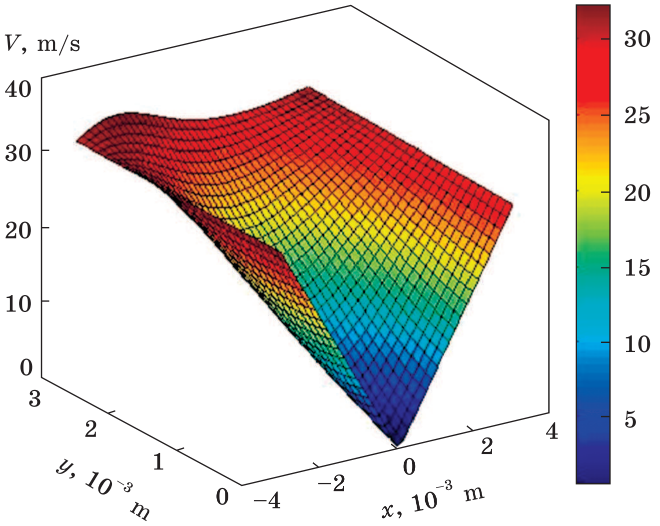

Fig. 1. The field of particle velocity during flow around an abrasive grain [13]

where a is a hyperbole parameter, semi-major axis, e is an eccentricity of hyperbola, α is an angle of rotation ($\alpha = -\gamma/2$, γ is cutting angle), V_0 is a cutting speed, and coordinate x_0 determines the plastic flow beginning, dependences were obtained for the calculation of deformation constituents, strain rates and their intensities:

$$e_{xx} = \int \varepsilon_{xx} dt\,,\; e_{yy} = \int \varepsilon_{yy} dt\,,\; e_{xy} = \int \varepsilon_{xy} dt, \tag{23}$$

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \varepsilon_{yy}^2 + \varepsilon_{xx}^2 + \frac{3}{2}\varepsilon_{xy}^2}\,, \tag{24}$$

$$e_i = \frac{\sqrt{2}}{3}\sqrt{(e_{xx} - e_{yy})^2 + e_{yy}^2 + e_{xx}^2 + \frac{3}{2}e_{xy}^2}\,. \tag{25}$$

These dependences make it possible to calculate the energy dissipation function, which can be expressed through the intensity of stresses and the strain rates by the formula

$$E = \sigma_i \varepsilon_i. \tag{26}$$

To determine the deformation power, the energy dissipation function must be integrated over volume:

$$N = \iiint\limits_{v} E dv, \tag{27}$$

where σ_i is the intensity of stresses, ε_i is the intensity of strain rates.

The stress intensity for different deformed media is a complex function of the strain intensity, strain rate, temperature, time, and other parameters.

Metals and alloys are a group of materials that are strengthened during plastic deformation. The stress intensity is a function of the strain intensity only:

$$\sigma_i = \sigma_m \varepsilon_i^n, \tag{28}$$

where n is the index of strain hardening; e_i is intensity of deformation; σ_m is the yield strength.

An example is considered: the operation of grinding a part of an alloy BT3-1, the speed of rotation of the abrasive wheel V_{wh} = 30 m/s, the speed of rotation of the workpiece V_w = 30 m/min, cutting depth t = $= 0.02 \cdot 10^{-3}$ m. Parts were machined with an abrasive wheel 63C40CM2K with geometry $\gamma = \alpha$ = 101.6 degrees. The velocity field of particle displacements for this one is shown Fig. 1.

The calculation of the deformation power according to the above algorithm gives the following result: $N \approx 9.93$ W. Below, there are the experimental investigation of the total cutting forces, it which converting to the single grain power give good agreement with theoretical calculations.

Therefore, sequential determination of main parameters during plastic deformation of metal based on the initial velocity field allows in long run to reach an important energy–power characteristic of any processing process, namely, the work of plastic deformation [14]. As another important advantage of method, it should be noted that flow kinematics can be described at any strain rates and wedge angles, which along with the application area, allows us to conclude that proposed research technique is universal. The influence of strain rate, angle wedge, characteristics of processed material and other parameters of processing modes will affect shape and boundary of elastoplastic zone. It is known, as we move away from wedge, which is perturbation source of the deformation waves, the plastic metal flow rates decay to zero in region of unstrained volumes. Thus, the mechanism, determining coordinate, *i.e.*, the beginning metal plastic flow, remains an open question in the developed theory. It can be concluded that determining nature of attenuation rate of plastic flow metal, as well as the boundaries of zone of elastoplastic deformation is a key and rather complex issue requiring additional research.

3. Theoretical Investigation of the Stress–Strain State in the Cutting Zone

To determine the stress–strain state in an elastic half-space with a boundary half-plane arising under the action of normal and tangential forces applied in a closed region, the classical approach proposed by Boussinesq and Cerruti using the theory of potential and the well-known solution of the problem of elasticity is commonly applied [15, 16]. As known, solution of most applied problems *via* the classical approach, presents certain difficulties and shortcomings. First of all, the solution obtained by Boussinesq gives satisfactory results for the elastic region of deformation the metal; dependence (29) gives overestimated values:

$$\sigma = \frac{2P\sin(\alpha)}{\pi h}, \tag{29}$$

where α is the angle between the direction of action of the force $\mathbf{P}$ ($|\mathbf{P}| = P$) and the radius-vector of the point $\mathbf{h}$ ($|\mathbf{h}| = h$) in question.

One of the significant problems is the preliminary determination the components of cutting forces functionally related the stresses acting in the cutting region. In case of the study of kinematics plastic flow of deformed material in the cutting zone and determination of energy–power descriptions of process, we have to define the field of speeds as the basic data. For that construction, in turn necessary, boundary conditions are needed in the form of coordinates the plane beginning of the plastic flow of the metal, which directly depends on the size of the elastoplastic zone. On this account, again the preliminary calculation of cutting forces is nevertheless needed for the solution of the objectives. In our opinion, the most rational method for solving tasks is an express calculation of cutting forces using well-known engineering methods for calculating projections of cutting forces. It is obvious that the use of empirical dependences to calculate the cutting forces is quite laborious and requires a significant array of empirical research on each material. For this reason, most researchers conducted surveys to establish theoretical equations linking the components of the cutting forces with physical-mechanical characteristics of the processed material, geometrical parameters of the tool and the dimensions of cut layer [17, 18]. Among the whole variety of analytical dependences obtained by various authors, the Zorev's equations [18] give the most complete account of the deformation mechanism during cutting and high accuracy. These equations were obtained based on the hypothesis of equality of shear stresses during cutting and compression as well as tension under equal deformations, and the Rosenberg methodology [17], based on hypothesis of the equality specific work of plastic deformation with equality of deformations. Zorev's equations [18] showed a good coincidence with the experimental values of cutting forces. Both methods give satisfactory results, which are confirmed by numerous experimental researches. In addition, the choice of method is aimed at determining only the boundaries of the region of the stress–strain state, so, you can choose one of the known methods, which is most convenient for the researcher. It depends on the available data very often. It is also possible to use empirical dependences to calculate the sought-for components.

In Ref. [16], author obtained formulas for determining normal stress σ_z acting on the side of cutting edge of the cutter and maximum pressure on the back surface of the tool σ_y:

$$\sigma_z = \frac{P_z}{BC\cos\gamma}\left(\frac{2C}{a\varsigma(\mu_f + \mathrm{tg}(\Phi - \gamma))} - 1\right),$$
$$\sigma_y = \frac{P_y}{BC_b\cos\alpha}\left(\frac{2C}{a\varsigma(\mu_b + \mathrm{tg}(\Phi - \gamma))} - 1\right), \quad (30)$$

where B is the width of the plates; γ and α are front and back angle of the cutter; a is thickness of the cut layer; ς is shrinkage of plates; μ_f (μ_b) is coefficient of friction on the front (back) surface; C (C_b) is the contact length along the front (back) surface; Φ is the angle, which determines the direction of shear during cutting.

The set of Eqs. (30) makes possible to determine geometric parameters of deformable cutting zone, such as the length of the leading cold-hardening in front of the cutter tip in cutting direction and cold-hardening depth of surface layer under the treated surface.

Starkov [16] has obtained expressions for determining the dimensions of the plastically deformable cutting zone

$$l = \frac{1}{g}\ln\frac{\sigma_z}{\sigma_{0.2}}, \quad H = \frac{1}{g}\ln\frac{\sigma_y}{\sigma_{0.2}}, \quad (31)$$

where $\sigma_{0.2}$ is a yield stress, and g is a distribution index.

Distribution index g linearly depends on the relative content in heat-resistant alloys of the hardening phase and is essentially a characteristic of the material.

The results of experimental investigation carried out by Starkov [16] showed that free cutting is accompanied by formation of an area advanced hardening in front of cutter and a hardened area of metal under the treated surface.

Thus, length of zone of leading work-hardening is on average 2–3 times greater than depth of work-hardening surface layer of the machined part (Table 1). The zone of advanced hardening moves in front of the moving cutting wedge at a speed equal to collective or average speed movement of dislocation ensemble, which consists dislocations of different types and signs.

Figure 2 shows calculated and experimental values dimensions of the plastically deformed cutting zone obtained by decorating after broaching three heat-resistant alloys.

The experimental values of l and H are shown for two cutting speeds: 4 and 7.9 m/min for ХН56ВМКЮ and ЖС6КП alloys, 4 and 22.5 m/min for ХН77ТЮР alloy; for higher speeds the values of l and H are marked by shaded points. Data analysis in Fig. 2, carried out by the author [16],

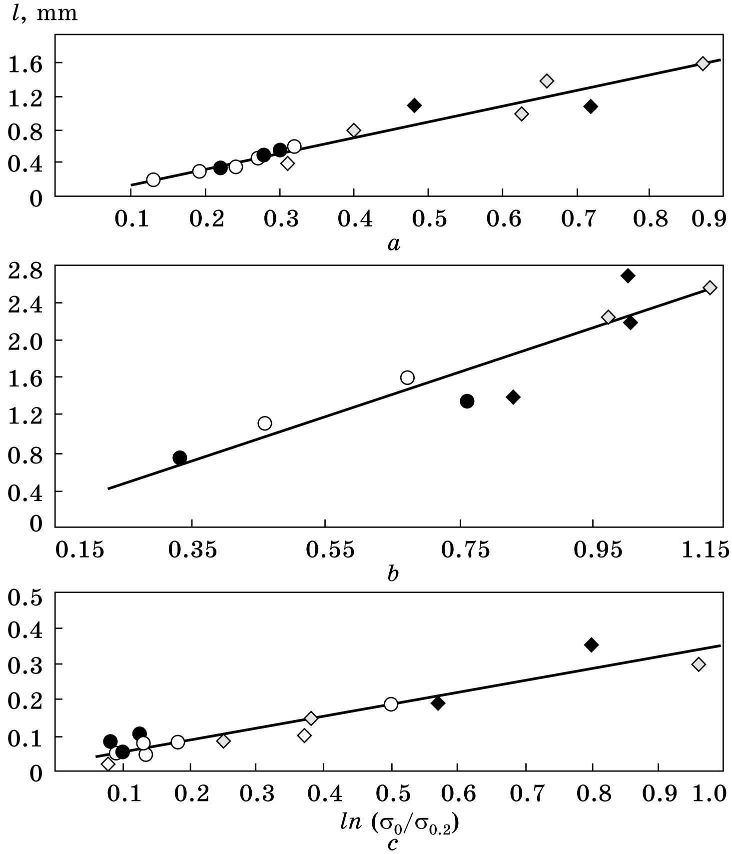

Fig. 2. Comparison of calculated (◊, ○) and experimental (●, ♦) values of the depth of hardening H and the length of leading hardening l with free cutting of ЖС6КП (*a*), ХН77ТЮР (*b*), and ХН56ВМКЮ (*c*) alloys [16]

showed that the experimental values of l and H are fairly tightly grouped around the communication line between the size of the reinforcing zone and the logarithm of the ratio of the effective (calculated) stress to the yield strength (reference). Thus, it is possible to conclude that formulas (31) exactly enough determine size the plastic deformed zone, binding them to module of work-hardening processed material and its structural state through the index of g. It is possible to suppose that formulas are suitable for description sizes of the plastic deformed cutting zone not only of heat-resistant alloys, but also of any other metals and of alloys and will differ only in the coefficient value.

Let us consider in more detail the process of plastic deformation. It is well known that plastic deformation and cold-hardening of a metal is a consequence of the nucleation and motion of dislocations newly appearing or already existing in the metal. During deformation, dislocations

Table 1. **The results of measuring sizes of plastic deformed zone at the free cutting**

Processed material	Cutting conditions		Sizes of the plastic deformed zone		Increment of depth of the hardened layer during secondary deformation	
	v, m/minute	t, mm	in front of cutter	under the cutter	mm	%
Nickel alloy XH77TЮP	4.00	0.14	2.29	1.16	0.14	12.00
	4.00	0.34	2.79	1.58	0.15	9.50
	22.50	0.14	1.41	0.68	0.02	2.90
	22.50	0.46	2.85	1.27	0.11	8.70
	4.00	1.91	6.12	3.15	—	—
	22.50	0.54	2.20	2.0	—	—
Electrical engineering steel 1511	4.00	0.32	1.50	0.81	0.05	6.20
	4.00	0.38	1.60	0.94	0.04	4.30
	22.50	0.07	1.20	1.10	0.30	27.30
	22.50	0.28	—	1.15	0.07	6.10

are the first to move in the slip system where the tangential stresses reach a maximum, and the plastic flow of the metal is possible only if the tangential stresses exceed a certain critical threshold value close to the yield strength [16]. The magnitude of this threshold stress depends on the initial dislocation structure, that it is determined by the initial dislocation density, which in turn is determined by the type of crystal lattice, the presence of impurities and technological heredity in an aspect that includes all previous operations of the technological process. The motion of dislocations, as well as other defects is accompanied by dissipation of the strain energy with its transition to heat. Thus, we make conclude that the heating processes associated with the deformation of the metal, as well as the propagation of zones of elastic-plastic deformation and heat are interconnected and apparently require a comprehensive review.

4. The Mechanism of Inhibition of Dislocations and Transformation of Energy at the Deformation

The reference [19] presents the results of experiments performed by the author in order to determine the thermal conductivity of a metal subjected to advanced plastic deformation (APD). Based on the analysis of experimental data, a reduction in the thermal conductivity of deformed metal was established as compared with the material not subjected to preliminary plastic deformation [19]. Thus, with the increase of work-hardening depth, the total heat conductivity of sample decreases, which allowed concluding the presence of the phenomenon of dynamic thermal conductivity of the workpiece surface layer, at that, with a change in

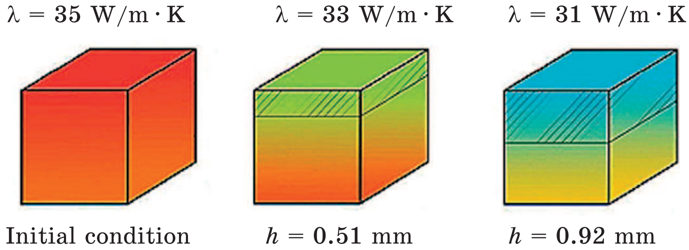

Fig. 3. Values of the integral thermal conductivity of the sample depending on the depth of the deformed metal layer [20]

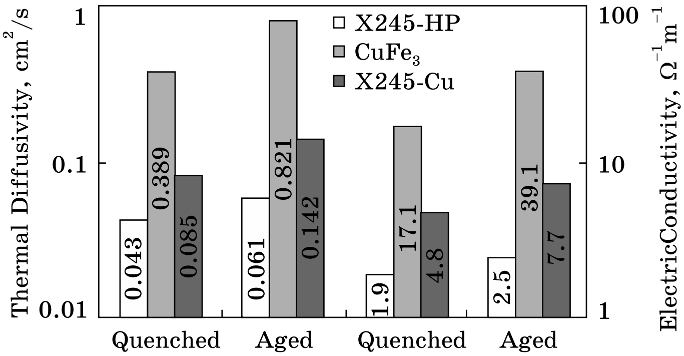

Fig. 4. Thermal diffusivity and electrical conductivity of the X245 steel, X245–Cu composite, and $CuFe_3$ alloy in two states: the quenched state as well as quenched and aged state [21]

the depth of the leading plastic deformation, the thermal conductivity in the riveted layer changes [20]. However, at calculating the thermal conductivity of the riveted layer, it was found that with an increase in the depth of the hardened layer, an increase in the value of its thermal conductivity is observed. The author explains the reason for this by the nonlinear distribution of deformation along the depth of the riveted layer with a maximum near the surface (at a depth of 0.5 mm, the deformation propagates more intensively as compared to a depth in 0.92 mm) (Fig. 3).

Heat conduction resistance, as known, is the result of a destruction periodicity of the crystal lattice. These imperfections are associated with vacancies, dislocations, and other defects in the crystal structure. Plastic deformation leads to cold-hardening, which is a consequence of the advancement and nucleation of new dislocations and vacancies. New distributions and the former certainly have influence on heat conductivity of the riveted layer, but appearance of vacancies in more considerable degree results in the increase of dispersion of electrons that hampers the transmission of energy in turn. The influence of cold-hardening on the thermophysical characteristics of the material is also confirmed in Ref. [21]. Heat treatment leads to a change in the structure, mechanical properties of the density of dislocations and thermophysical properties, which is reflected in the graph in Fig. 4.

The paper [22] presents a methodology for calculating the activation energy of dislocation motion and the activation volume. It is well known

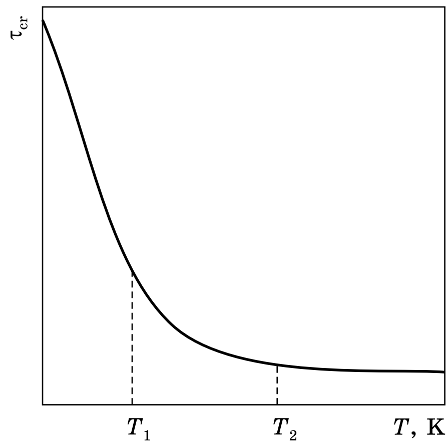

Fig. 5. Critical shear stress vs. temperature (schematic dependence) [22]

that the resistance to the motion of dislocations in crystals is due to the existence of barriers, overcoming of which is either athermal in nature, or can be facilitated by thermal fluctuations [23–30]. Barriers of the first type include, *e.g.*, long-range stress fields, grain boundaries, particles of other phases; barriers of the second type include Peierls–Nabarro barriers, 'forest' dislocations, and thresholds on screw dislocations. A critical shear stress is a function of temperature and strain rate: $\tau_{cr} = \tau_a + \tau_T(T, \varepsilon)$.

Currently, there are two widely used approaches to the experimental determination of parameters characterizing the temperature-dependent part of the flow stress, the activation energy of dislocation motion (U_0) and the activation volume (V). This is an approach developed by Seeger and Conrad, based on the equation for the strain rate as the rate of a thermally activated process [23–26], and the approach developed by Milman and Trefilov, based on the analysis of the dependence of the critical shear stress τ_{cr} on temperature [27, 28]. The authors noted the shortcomings of the Seeger–Conrad method approach; in particular, low accuracy of calculations at low stresses and strains. These shortcomings were successfully overcome in the integrated model of Trefilov and Milman combining the approaches of Seeger and Haasen.

A typical temperature dependence of the critical shear stress τ_{cr} is shown schematically in Fig. 5. At temperatures below T_1, the dependence $\tau_{cr}(T)$ is described by a linear equation, and in the temperature range $T_1 < T < T_2$, it is exponential. Above T_2, up to temperatures $(0.35–0.4)T_{melt}$, hardening is athermal in nature and is not determined by barriers, which can be overcome by thermal fluctuations.

A useful feature of the Trefilov–Milman approach is the fact that thermally activated parameters can only be determined using the temperature dependence of the critical shear stress (or the temperature dependence of the flow stress) without varying the strain rate. The authors note that thermal activation analysis of critical shear stress τ_{cr} performed by the proposed method leads to the dependence of the activation energy and activation volume on the stress, while these values determined by the Trefilov–Milman method are material constants depending on the type of interatomic bond, nature of the potential barrier [27, 28].

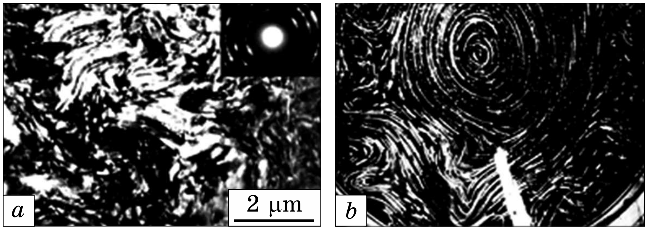

Fig. 6. Structure of Fe–17Ni–10W–10Co–1Mo–1Ti alloy past 87% hydro-extrusion (*a*) and its analogy with convection of liquid (with Al particles) in rotating cylinder, heated at the external surface and cooled to the centre (*b*) [33]

The mechanism for converting the kinetic energy of a moving dislocation into thermal is as follows: the elastic field of a moving dislocation disturbs equilibrium of phonon gas, which leads to the outflow of energy from dislocation to phonons with effective braking of dislocations and excitation of increasing number of phonon modes [31]. In connection with the foregoing, we need to link geometric parameters of elastoplastic zone, the plastic energy of dislocation, and thermophysical characteristics of the material.

We apply dimensional theory to the problem. Above, we partially carried out a schematization of the phenomenon considered in a metal, identified the factors and quantities of interest to us [32]. Thus, the size of plastic zone deformation depends on speed of dislocations, a certain characteristic size and thermal characteristics. Then, we can write $l = f(V, S, a)$. The measurement of speed, area, and thermal diffusivity were chosen as the basic units.

From the four parameters, it is possible to form one independent dimensionless combination $(VS/(la))$ or mutually inverse to it, where V is dislocation sliding velocity, S is characteristic area, l is characteristic size of the deformable region, a is thermal diffusivity.

The paper [33] reports on the analysis of structures resulting from severe plastic deformations, which allowed authors to suggest possibility of layered flow of deformed body under condition of continuity between layers. Indeed, metal layers can move at different speeds due to friction the surface of sample with the matrix due to a decrease in movement speed from centre to the surface, as well as due to the temperature gradient (heated sample in a less warm or cold matrix) (see Fig. 6). The formation of vortices can occur in collision of flows with different speeds. Such vortices are similar to whirlpools; they are modelled as $3D$ formations in columns form or cylindrical regions that rotate, bend, and combine. The similarity described above may be useful for the model description of severe plastic deformation from a new point of view, which began in the work of Beygelzimer [34]. A similar hypothesis was developed further in our study [1] and, as will be shown below, makes it possible to look differently at the process of metal deformation and to link the thermophysical and mechanical properties of the material.

5. Dislocation–Kinetic Approach

The deformation process can be considered as a transport phenomenon, as a result of which there is spatial energy transfer, where dislocation can be considered as a carrier of traditional shear. In this case, the dislocation will act as an elementary carrier of strains, namely quantum of deformation. With this approach, the dislocation can be considered as a quasi-particle. For such a dislocation gas, it can be applied the molecular-kinetic method of investigation, which in our case should be called the dislocation–kinetic method. This allows us to calculate the heat flux in such a quasi-gas using the kinetic theory of gases. If the temperature of such gas is different in different places, then, the average dislocation energy will also be different in different places, then, by analogy with the formulas [35, 36], for dislocation gas, can write formula that allows calculating thermal conductivity (coefficient):

$$\lambda = \frac{1}{3}\langle V\rangle\langle L\rangle c\rho\,, \tag{32}$$

where $\langle V\rangle$ is an average dislocation sliding velocity, $\langle L\rangle$ is an average length of dislocation line running, c is a specific heat capacity, ρ is a metal density. If we go to the sources of the derivation of formula (32) for gases and phonons, then, we can go to a single elementary site perpendicular to the direction of dislocations motion $\langle V\rangle$. We write the well-known equation for the heat flux through the elementary side:

$$q = -\frac{1}{6}\langle V\rangle nS\frac{i}{2}k_B\frac{dT}{dz}2\langle L\rangle, \tag{33}$$

where n is number of gas molecules; i is sum of the number of translational, rotational and doubled number of vibrational degrees of molecules freedom; dT/dz is projection of the temperature gradient on z axis; S is surface perpendicular to the dislocations movement and equivalent to the area swept by dislocations; k_B is Boltzmann constant. Taking into account equality $(i/2)nk_B = \rho c$ and the Fourier law for thermal conductivity $q = -\lambda(dT/dz)S$, we obtain:

$$\frac{\lambda S}{\langle L\rangle} = \frac{1}{3}\langle V\rangle S\rho c \tag{34}$$

or

$$\frac{aS}{\langle L\rangle} = \frac{1}{3}\langle V\rangle S. \tag{35}$$

We hypothesized that the area in formula (35) is equivalent to the surface area swept by moving dislocations (Fig. 7). There were two rea-

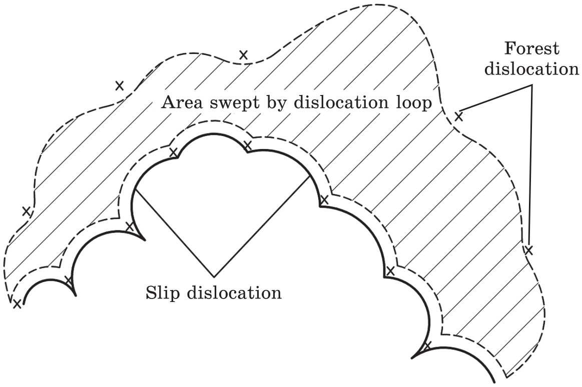

Fig. 7. Dislocation motion

sons for constructing such a hypothesis. First, dislocations in a metal have different lengths and random orientations that causes them to slip in process of deformation in different directions covered by a hemisphere with a base plane perpendicular to the direction of strain rate. Secondly, there is a correlation between deformation and area swept by dislocations during deformation. The first argument in favour of developed theory makes it possible to consider the surface S as directly proportional to the area swept, and taking into account the priority of sliding direction in the load direction allows us to remove the 1/3 factor from formula (35) and assume the equivalence S of the area swept by dislocations. The Arrhenius-type equation for the plastic strain rate confirms the second argument about relationship between deformation and the area swept by dislocations [37]:

$$\dot{\varepsilon} = NA\,b\nu_0 \exp\left(-\frac{\Delta g - \tau_{\text{ef}}\,\upsilon}{k_B T}\right),$$

where N is number of dislocations per unit volume, A is area swept by dislocations, b is Burgers vector module, ν_0 is frequency response depending on the nature of the obstacle and method of it overcoming, g is free energy change due to local atomic displacements during activation (equivalent to the Helmholtz free energy).

Based on the above reasoning and analysis of research results obtained by other authors, we can transform dependence (35) and use the result for further calculations.

If vector $\mathbf{z}$ (with $|\langle \mathbf{z}\rangle| = S/\langle L\rangle$) is aligned with the dislocation slip velocity vector, the modulus of this vector is just the length of leading hardening zone. Consider the resulting physical quantity $\langle V\rangle S/|\langle \mathbf{z}\rangle|a$.

We turn to the formulas obtained by Starkov [16], replace the parameter g^{-1} with parameter $\langle V\rangle S/a$ in formula (31), then, we obtain:

$$\left[\ln\frac{\sigma_z}{\sigma_{0.2}}\right]^{-1} = \frac{\langle V\rangle S}{a|\langle \mathbf{l}\rangle|}, \quad \left[\ln\frac{\sigma_y}{\sigma_{0.2}}\right]^{-1} = \frac{\langle V\rangle S}{a|\langle \mathbf{h}\rangle|}.$$

Let us analyse the obtained dimensionless expression $\langle V\rangle S/|\langle \mathbf{z}\rangle|a$. The value in the numerator $\langle V\rangle S$ is the growth rate of the deformable region volume in the direction of deformation, and the denominator is the growth rate of region volume bounded by isothermal surface, respectively (Fig. 8). The obtained dimensionless coefficient is a cross between the modified Péclet and Fourier criteria and describes convective heat transfer. On the other hand, the obtained complex can be considered as a criterion for the homochronism of the dissipation energy of plastic deformation and the rate of restructuring temperature field within the deformable region. If we conduct further analysis, it is possible really to find confirmation of this definition. Therefore, the relations (S/a) and $(V/|\mathbf{z}|)$ represent nothing more than thermally characteristic and deformation time, respectively. Denote the expression $\langle V\rangle S/|\langle\mathbf{z}\rangle|a$ by analogy with the Péclet number symbol Df. So we can write $Df = [\ln(\sigma_z/\sigma_{0.2})]^{-1} = \langle V\rangle S/(a|\langle\mathbf{l}\rangle|)$.

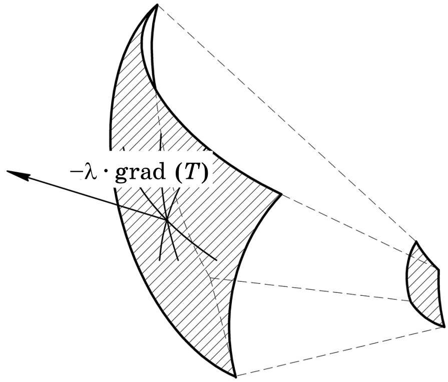

Fig. 8. Region bounded by isothermal surface

We consider the deformation number Df, which we introduced. The formula has a physical sense only when $\sigma \geq \sigma_{0.2}$, *i.e.* the plastic flow is realized.

As the stress acting in machining zone increases, the number Df decreases, which corresponds to regular growth of vector module $|\langle\mathbf{z}\rangle|$, at other constant data. Thus, large Df numbers correspond to initial degree of plastic flow and less developed plastic deformation. When $\ln(\sigma_z/\sigma_{0.2}) = 1$, the number $Df = 1$ that corresponds to the current stress equal to $\sigma \approx 2.718\sigma_{0.2}$; then, $Df \in (0; +\infty)$. In the previously considered cases (Fig. 1), the maximum values of Df do not exceed 20 units, which is consistent $\ln(\sigma_z/\sigma_{0.2}) = 0.05$.

Let us analyse the obtained result. The comparison of our formula $Df = \langle V\rangle S/(a|\langle\mathbf{l}\rangle|)$ and $l = (1/g)\ln(\sigma_z/\sigma_{0.2})$ results to $g^{-1} = \langle V\rangle S/a$, and therefore $\langle V\rangle S/a$ is invariant to the conditions of deformation and thermophysical characteristics of material. In experimental investigation, it is confirmed that [19, 21] thermophysical properties of materials change after deformation, which is explained by an increase in number of defects in the riveted material. At the same time, a change in substructure of material and related changes in properties cannot significantly change the coefficient of thermal diffusivity, and it compensates for the wide range of changes in sliding dislocations velocity $\langle V\rangle$ observed during deformation. Then we can conclude that product $\langle V\rangle S$ remains constant.

Show it. It is known that the stress of the flow of material during deformation obeys the law $\sigma \approx \sqrt{\rho}$, which was first introduced by Taylor [16, 38], and area swept by dislocations can be determined by the formula $S = 1/\rho$, then $\sigma \approx 1/\sqrt{S}$. It is also known that sliding dislocations

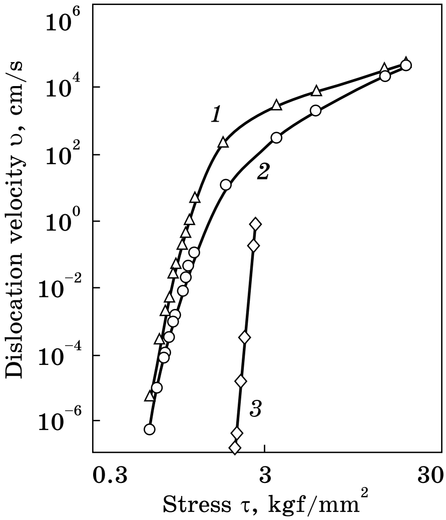

Fig. 9. Mobility of dislocations in crystals, where *1* — edge dislocations at 300 K *2* — screw dislocations at 300 K, *3* — screw dislocations at 77 K [40]

velocity depends on the stress of the metal flow (Fig. 9) and obeys the relation [39]:

$$V = k\sigma^m, \tag{36}$$

where k is a material constant.

Since the value of m can vary from 1 to 100, we can write Eq. (36):

$$k = VS^n, \tag{37}$$

where $n \in [0.5; 50]$.

One way or another, the growth of one of the parameters, namely, the velocity or area with dislocations, entails a decrease in the other one according to the law (37); in the case of $n = 1$, the connection between V and S is inversely proportional, which only confirms our results.

Let us estimate the value of leading zone hardening according to the formula $|\langle \mathbf{z} \rangle| = S/\langle L \rangle$. It is known that in annealed polycrystals dislocation density varies in the range $\rho = 10^{10}–10^{12}$ m^{-2}. Commonly, the distance between obstacles is assumed to be $\langle L \rangle \approx 1/\sqrt{\rho}$.

The area swept by dislocations varies from $1/\rho$ to d^2 (grain size), therefore, the limits of its change can be estimated [38]. After a substitution, we obtain $S / \langle L \rangle = (1 / \rho)\sqrt{\rho} = 1/\sqrt{\rho}$, $|\langle \mathbf{z} \rangle| = 10^{-5}–10^{-6}$ m. However, more accurate estimate requires taking experimental data on certain material. Therefore, it is known [16] that for samples of heat-resistant (annealed) alloys, the distance between the slip bands was within $\langle L \rangle = 2.54 \cdot 10^{-8}$ m, then,

$$|\langle \mathbf{z} \rangle|_{\max} = \frac{10^{-10}}{2.54 \cdot 10^{-8}} = \frac{10^{-2}}{2.54} = 0.0039 \text{ m},$$

$$|\langle \mathbf{z} \rangle|_{\min} = \frac{10^{-12}}{2.54 \cdot 10^{-8}} = \frac{10^{-4}}{2.54} = 0.000039 \text{ m},$$

which correspond to experimental data. The calculation of sizes of advanced riveting zone through the average grain size of the alloys ($d = 0.32–0.45$ mm) gives too high estimates, this is due fact that with this formulation of problem, the presence of stoppers on path of dislocation movement within individual grains is not taken into account, and only their borders are considered as obstacles. In this case, the area swept by dislocations is on average five orders of magnitude larger, which gives overestimated data when calculating the boundaries of the elastoplastic zone. It should be understood that such calculations are only evaluative in nature and, due to wide range of changes in dislocation density, cannot be used to determine the desired boundaries. In our opinion, the most rational is empirical definition of $\langle V\rangle S/a = 1/g$ for a certain material.

6. Conclusions

The bases of theoretical analyses of technological processes in multidimensional spaces are considered. Analysis of the deformed state of material of a part during diamond smoothing and grinding is made using the general equations of continuum mechanics and displacement velocity fields. We established that determination of boundary of elastoplastic deformation zone, and the methods for its determination are important theoretical issues in the field of deformation mechanics. The methods of determining the size of plastic zone during machining by cutting are considered. Analysis of various methods for calculating cutting forces that can be used for calculation of power characteristics of machining process and determination of the stresses acting in cutting zone is presented. The Starkov's model [16] for determination of boundaries of elastoplastic zone was further developed, based on which a phenomenological model was proposed that reveals the physical meaning of coefficient in formulas for calculation of the boundaries of zone of plastic flow metal. A dislocation–kinetic method is developed for determination of boundaries of the elastoplastic deformation zone, based on the concept dislocation as a quasi-particle, which is a strain quantum. Using the dislocation–kinetic model, a new material characteristic is introduced, namely, the deformation number (similarity criterion), which relates dissipation of plastic strain energy and rate of temperature field tuning.

REFERENCES

1. M.O. Kurin, *Metallofiz. Noveishie Tekhnol.*, **40**, No. 7: 801 (2018). https://doi.org/10.15407/mfint.40.07.0859
2. L.B. Zuev, S.A. Barannikova, and A.G. Lunev, *Prog. Phys. Met.*, **19**, No. 4: 379 (2018). https://doi.org/10.15407/ufm.19.04.379
3. Yu.V. Milman, S.I. Chugunova, I.V. Goncharova, and A.A. Golubenko, *Prog. Phys. Met.*, **19**, No. 3: 271 (2018). https://doi.org/10.15407/ufm.19.03.271
4. A.I. Dolmatov, S.V. Sergeev, M.O. Kurin, V.V. Voronko, and T.V. Loza, *Metallofiz. Noveishie Tekhnol.*, **37**, No. 7: 871 (2015). https://doi.org/10.15407/mfint.37.07.0871
5. Yu.N. Alekseev, *Vvedenie v Teoriyu Obrabotki Metallov Davleniem, Prokatkoy i Rezaniem* [Introduction to the Theory of Metal Processing via the Pressure, Rolling and Cutting] (Kharkov: Izd-vo KhGU, 1969) (in Russian).
6. A.A. Kabatov, *Issues of Design and Manufacture of Flying Vehicles*, No. 1 (73): 67 (2013) (in Russian).

7. A.A. Kabatov, *Tekhnologiya Almaznogo Vyglazhivaniya Detaley Aviatsionnykh Dvigateley i Agregatov* [Technology for Diamond Smoothing of Aircraft Engine Parts and Units] (Thesis of Disser. for PhD) (Kharkiv: National Aerospace University 'Kharkiv Aviation Institute': 2014) (in Russian).
8. A.I. Dolmatov, A.A. Kabatov, and M.A. Kurin, *Aerospace Technic and Technology*, **100**: 12 (2013).
9. A.I. Dolmatov, A.A. Kabatov, and M.A. Kurin, *J. Mechanical Engineering NTUU 'Kyiv Polytechnic Institute'*, **67**: 186 (2013) (in Russian).
10. A.I. Dolmatov, A.A. Kabatov, and M.A. Kurin, *Metallofiz. Noveishie Tekhnol.*, **35**: 1407 (2013).
11. Yu.N. Alekseev, V.K. Borisevich, and P.I. Kovalenko, *Impul'snaya Obrabotka Metallov Davleniyem*, **5**: 112 (1975).
12. A.G. Odintsov, *Uprochneniye i Otdelka Detaley Poverkhnostnym Plasticheskim Deformirovaniyem* [Hardening and Finishing of Parts by Surface Plastic Deformation] (Moscow: Mashinostroyeniye: 1987) (in Russian).
13. S.M. Nizhnik, *Tekhnologiya Shlifovaniya Detaley Aviatsionnykh Dvigateley s Uchetom Uvelicheniya Aktivnoy Poverkhnosti Abrazivnogo Zerna* [Grinding Technology of Aviation Engine Parts, Taking into Account the Increase of the Active Surface of Abrasive Grain] (Thesis of Disser. for PhD) (Kharkiv: National Aerospace University 'Kharkiv Aviation Institute': 2018) (in Russian).
14. M.O. Kurin and M.V. Surdu, *Metallofiz. Noveishie Tekhnol.*, **39**, No. 3: 401 (2017) (in Ukrainian).
https://doi.org/10.15407/mfint.39.03.0401
15. K.L. Johnson, *Contact Mechanics* (Cambridge University Press: 1985).
https://doi.org/10.1017/CBO9781139171731
16. V.K. Starkov, *Fizika i Optimizatsiya Rezaniya Materialov* [Physics and Optimization of Cutting materials] (Moscow: Mashinostroenie: 2009) (in Russian).
17. A.M. Rosenberg and A.N. Eremin, *Elementy Teorii Protsessa Rezki Metalla* [Elements of the Theory of Metal Cutting Process] (Moscow: Mashgiz: 1956) (in Russian).
18. N.N. Zorev, *Issledovanie Elementov Mekhaniki Protsessa Rezaniya* [The Study of Elements of the Mechanics of the Cutting Process] (Moscow: Mashgiz: 1952) (in Russian).
19. A.A. Bondarev, *Issledovanie Vliyaniya Operezhayushchey Plasticheskoy Deformatsii na Effektivnost' Protsessa Rezaniya Konstruktsionnykh Staley* [Study of the Influence of Leading Plastic Deformation on the Effectiveness of the Cutting Process of Structural Steels] (Thesis of Disser. for PhD) (Volgograd: Volgograd State Technical University: 2016) (in Russian).
20. A.A. Bondarev, Y.N. Oteniy, Yu.L. Chigirinsky, Yu.N. Polyanchikov, D.V. Krainev, and D.V. Pronichev, *Izvestiya VSTU. Ser. Advanced Technology in Machine Building*, **173**: 7 (2015) (in Russian).
21. S. Klein, S. Weber, and W. Theisen, *J. Mater. Sci.*, **50**: 3586 (2015).
https://doi.org/10.1007/s10853-015-8919-y
22. S.O. Firstov and T.G. Rogul, *Metallofiz. Noveishie Tekhnol.*, **39**, No. 1: 33 (2017) (in Russian).
https://doi.org/10.15407/mfint.39.01.0033
23. A.H. Cottrell, A. Seeger, and J.L. Amorys, *Deformation and Flow of Solids / Verformung und Fliessen des Festkųrpers. International Union of Theoretical and Applied Mechanics / Internationale Union für Theoretische und Angewandte Mechanik* (Ed. R. Grammel) (Berlin–Heidelberg: Springer: 1956), pp. 33–52.
https://doi.org/10.1007/978-3-642-48236-6_5

24. A. Seeger, *Kristallplastizitat* [Crystal Plasticity] (Berlin: Springer-Verlag: 1958) (in German).
https://doi.org/10.1007/978-3-642-45890-3_1
25. H. Conrad, *Acta Met.*, **6**, No. 5: 339 (1958).
https://doi.org/10.1016/0001-6160(58)90071-3
26. H. Conrad, *Tekuchest i Plasticheskoe Techenie OTsK-Metallov pri Nizkikh Temperaturakh. Struktura i Mekhanicheskie Svoystva Metallov* [Yield and Plastic Flow for B.C.C. Metals at Low Temperatures. Structure and Mechanical Properties of Metals] (Moscow: Metallurgiya: 1967) (Russian translation).
27. Yu.V. Mil'man and V.I. Trefilov, *O Fizicheskoy Prirode Temperaturnoy Zavisimosti Predela Tekuchesti. Mekhanizm Razrusheniya Metallov* [On the Physical Nature of the Temperature Dependence of Yield Stress. Metal Fracture Mechanism] (Kiev: Naukova Dumka: 1966) (in Russian).
28. V.I. Trefilov, Yu.V. Milman, and S.A. Firstov, *Fizicheskie Osnovy Prochnosti Tugoplavkikh Metallov* [Physical Bases of the Strength of Refractory Metals] (Kiev: Naukova Dumka: 1975) (in Russian).
29. P. Haasen, *Acta Met.*, **5**, No. 10: 598 (1957).
https://doi.org/10.1016/0001-6160(57)90129-3.
30. P. Haasen, *Mekhanicheskie Svoistva Tverdykh Rastvorov i Intermetallicheskikh Soedineniy. Fizicheskoe Metallovedenie* [Mechanical Properties of Solid Solutions and Intermetallic Compounds. Physical Metallurgy] (Eds. R.W. Cahn and P. Haasen] (Moscow: Metallurgiya: 1987) (Russian translation).
31. V.I. Al'shits and V.L. Indenbom, *Sov. Phys. Usp.*, **18**, No. 1: 1 (1975).
https://doi.org/10.1070/PU1975v018n01ABEH004689
32. L.I. Sedov, *Metody Podobiya i Razmernosti v Mekhanike* [Similarity and Dimension Methods in Mechanics] (Moscow: Nauka: 1977) (in Russian).
33. T.E. Konstantinova, *Fizika i Tekhnika Vysokikh Davleniy*, **19**: 7 (2009) (in Russian).
34. Ya.E. Beygelzimer, *Fizika i Tekhnika Vysokikh Davleniy*, **18**: 36 (2008) (in Russian).
35. I.V. Savel'yev, *Kurs Obshchey Fiziki, Mekhanika. Molekulyarnaya Fizika* [Course in General Physics, Mechanics. Molecular Physics] (Moscow: Nauka: 1982) (in Russian).
36. S.V. Izmaylov, *Kurs Ehlektrodinamiki* [The Course of Electrodynamics] (Moscow: Gos. Uch.-Ped. Izd-vo Min. Prosv. RSFSR: 1962) (in Russian).
37. D.G. Verbilo, *Ehlektronnaya Mikroskopiya i Prochnost' Materialov. Ser.: Fizicheskoye Materialovedeniye, Struktura i Svoystva Materialov*, **18**: 104 (2012) (in Russian).
38. Frank A. McClintock, and Ali S. Argon, *Mechanical Behavior of Materials* (Addison-Wesley Pub. Co.: 1966).
39. R.W.K. Honeycombe, *The Plastic Deformation of Metals* (London: Hodder & Stoughton General Division: 1968).
40. V.L. Indenbom and A.N. Orlov, *Sov. Phys. Usp.*, **5**, No. 2: 272 (1962).
https://doi.org/10.1070/PU1962v005n02ABEH003412

Received 09.12.2019;
in final version, 18.05.2020

М.О. Курін
Національний аерокосмічний університет ім. М.Є. Жуковського
«Харківський авіаційний інститут»,
вул. Чкалова, 17, 61070, Харків, Україна

ВИЗНАЧЕННЯ МЕЖ ПЛАСТИЧНОЇ ЗОНИ
ДЕФОРМУВАННЯ МЕТАЛУ ПРИ РІЗАННІ

Основною метою роботи є аналіз проблеми визначення меж пружньо-пластичної зони за різних методів оброблення деталів різанням. Розглянуто різні методики визначення сил різання за механічного оброблення зі зніманням стружки, а також підходи до визначення напружено-деформованого стану матеріалу. Розглянуто структуру комплексних теоретико-експериментальних досліджень енергосилових параметрів процесів механічного обробляння. Для теоретичного дослідження енергосилових параметрів процесів запропоновано метод розрахунку пластичного деформування металів, заснований на замкнутій системі рівнянь механіки суцільних середовищ. Одержано вирази, за допомогою яких можна відтворювати просторову картину розподілу деформацій у металі за діамантового вигладжування та шліфування, що дає змогу наочно уявити механізм деформування та спростити аналіз деформованого стану матеріалу. Встановлено функціональний зв'язок між потужністю деформування та параметрами режиму оброблення деталей за діамантового вигладжування та шліфування. Запропоновано метод експрес-розрахунку сил різання з використанням відомих інженерних методик. Проаналізовано експериментальні та розрахункові дані щодо визначення розмірів пластично-деформованої зони важкооброблюваних матеріалів. Детально розглянуто механізм гальмування дислокацій і перетворення енергії під час деформації, в результаті чого розроблено дислокаційно-кінетичний підхід, в основі якого лежить поняття про дислокацію як про квазичастинку, що представляє собою квант деформування. З використанням дислокаційно-кінетичного підходу розроблено математичну модель, яка уможливлює виконання розрахунку величини зони випереджувального зміцнення, що підтверджено порівнянням із експериментальними даними. Доопрацьовано модель Старкова, пояснено фізичний зміст коефіцієнта в формулах для розрахунку меж зон зміцнення. Введено новий критерій подібности, що зв'язує дисипацію енергії пластичної деформації та швидкість перебудови температурного поля.

Ключові слова: пружньо-пластична зона, сили різання, дислокаційно-кінетичний підхід, критерій подібности, дисипація енергії.

М.А. Курин
Национальный аэрокосмический университет им. Н.Е. Жуковского
«Харьковский авиационный институт»,
ул. Чкалова, 17, 61070, Харьков, Украина

ОПРЕДЕЛЕНИЕ ГРАНИЦ ПЛАСТИЧЕСКОЙ ЗОНЫ
ДЕФОРМИРОВАНИЯ МЕТАЛЛА ПРИ РЕЗАНИИ

Основной целью работы является анализ проблемы определения границы упругопластической зоны при различных методах обработки деталей резанием. Рассмотрена структура комплексных теоретико-экспериментальных исследований энергосиловых параметров процессов механической обработки. Для теоретического исследования энергосиловых параметров процессов предложен метод расчёта пластического деформирования металлов, основанный на замкнутой системе уравнений механики сплошных сред. Получены выражения, с помощью которых можно воспроизводить пространственную картину распределения деформаций в металле при алмазном выглаживании и шлифовании, что позволяет наглядно представить механизм деформирования и упростить анализ деформированного состояния материала. Установлена функциональная связь между мощностью деформирования и параметрами режима обработки деталей при алмазном выглаживании и шлифовании. Рассмотрены различные методики определения сил резания при механической обработке со съёмом стружки, а также подходы к определению напряжённо-деформированного состояния материала. Предложен метод экспресс-расчёта сил резания с использованием известных инженерных методик. Проанализированы экспериментальные и расчётные данные по определению размеров пластически деформируемой зоны труднообрабатываемых материалов. Подробно рассмотрен механизм торможения дислокаций и преобразования энергии при деформировании, в результате чего разработан дислокационно-кинетический подход, в основе которого лежит понятие о дислокации как о квазичастице, представляющей собой квант деформирования. С использованием дислокационно-кинетического подхода разработана математическая модель, которая позволяет производить расчёт величины зоны опережающего упрочнения, что подтверждено сравнением с экспериментальными данными. Доработана модель Старкова, объяснён физический смысл коэффициента в формулах для расчёта границ зон упрочнения. Введён новый критерий подобия, связывающий диссипацию энергии пластической деформации и скорость перестройки температурного поля.

Ключевые слова: упруго-пластическая зона, силы резания, дислокационно-кинетический подход, критерий подобия, диссипация энергии.