In the paper, a correlation between acoustic velocities $V$, elastic moduli $M$, and densities $\rho$, with surface tension $\sigma_m$, and work of adhesion $W_{ad}$ of different liquid metals on a given ceramic is studied and demonstrated. Simulation program is developed and used for scanning acoustic microscopy (SAM) under operating conditions, which favour the generation of acoustic waves. As found, for the given systems, all investigated acoustic parameters exhibit good dependences with both $\sigma_m$ and $W_{ad}$. Analysis and quantification of the results lead to the determination of semi-empirical formulas. The expressions are as follow: log $(V_i) = 0.49 \log (\sigma_m / \rho_{sm}) + B_i$, $M = A_m \sigma_m$, $W_{ad} = C_i V_i$, and $W_{ad} = \xi (M / \rho_{sm})^{1/2} + D_i$, where $A_i$, $B_i$, $C_i$, $A_m$, $\xi$, and $D_m$ are characteristic constants for velocities and elastic moduli, the subscripts $m$ relate to the elastic moduli (Young’s or shear ones), and $i = L$, $T$, $R$ — to the propagating longitudinal, transverse, and Rayleigh waves’ modes. The importance of the deduced formulas lies in the possibility of prediction of surface tension and work of adhesion of such metal/ceramic interfaces depending on the elastic and acoustic characteristics.

Keywords: surface tension, work of adhesion, acoustic velocities, elastic constants, ceramics, liquid metals, interfaces.

**Introduction**

The interfacial phenomena between metals and ceramics are one of interest subject in science and engineering. The performance of several technological applications such as ceramic metal bonding, metal–
cement joining, ceramic–metal matrix composites [1], thermal-barrier coatings (TBC) [2], hard TiN-coating [3], photovoltaic materials [4], and thin metal films on ceramic substrates [5] is directly linked to the nature of the metal/ceramic interfaces. The behaviour of this interfacial phenomenon is related directly to the nature of interfacial bonding between metal and ceramic [6]. The adhesion of the metal/ceramic system is the most important factor of all metal bonds. It is defined by the change in the free energies of two materials when they come into contact [7].

The work of adhesion $W_{ad}$, between liquid metal and ceramic substrate is given by Young–Dupré equation relating surface tension of molten metal above melting temperature $\sigma_m$ and measured equilibrium contact angle $\theta$ formed between deposited liquid metal and its ceramic substrate (see Fig. 1) [6]:

$$W_{ad} = \sigma_m (1 + \cos \theta).$$  \hspace{1cm} (1)

Various non-destructive techniques are established to characterize the metals-ceramic interfaces [8]. Scanning acoustic microscope (SAM) is one of the important tools for non-destructive determination of adhesion [9]. It can be used quantitatively (microanalysis) and qualitatively (imaging). The microanalysis mode is employed to characterize not only the elastic properties of materials but also the interfacial adhesion via propagation of acoustic wave’s measurement. It is possible when the interface lies in a plane $xy$ and disturbs the propagation of surface acoustic waves (SAWs) [9].

In this paper, a new acoustical approach has been proposed to interpret and estimate the surface tension and work of adhesion in the molten metal/ceramic systems; it shows that these parameters are determined simultaneously via acoustic velocities and elastic moduli according to the semi-empirical formulas.

**Methodology and Materials**

**SAM Technique**

Scanning acoustic microscope can be applied for a quantitative characterization of the interfacial adhesion via the investigation of acoustic material signature, $V(z)$. This analog signal received by transducer and focused by the position of the acoustic lens at the sample against the distance $z$, under an incidence angle with the reflected ones [10–12]. Its determination based on the calculation of the reflection coefficient $R(\theta)$. The $V(z)$ is the result of the several interferences of all the leaky wave
modes, such as leaky SAW, leaky pseudo-SAW, leaky surface-skimming compressional wave, leaky Lamb wave, and harmonic waves. However, only the velocity of leaky SAWs has been extracted from the $V(z)$ curves in microanalysis mode [13]. In effect of the operating conditions of the SAM, only one significant mode dominates all other leaky SAWs modes. Hence, the introduction of fast Fourier transformation (FFT) analysis of $V(z)$ is adopted for the directly determining of the acoustic velocities of materials [13].

**V(z) Calculation.** The most important quantitative method for elastic parameters determination, in particular, SAW velocities in scanning acoustic microscopy are acoustic material signatures, also known as $V(z)$, which are obtained by recording the output signal, $V$, as the distance, $z$, between the sample and the acoustic lens is varied. Such curves, that can be measured experimentally, can also be calculated theoretically, via the angular spectrum model [14], from the following expression:

$$V(z) = \int_{\theta_{\text{max}}}^{\theta} P^2(\theta) R(\theta) \exp(2jk_0z\cos\theta) \sin\theta \cos\theta d\theta. \quad (2)$$

Here, $P(\theta)$ is the distribution function, $k_0 = 2\pi/\lambda$ is the wave number in the coupling liquid, $j^2 = -1$, $\theta$ is the angle between the wave vector $k$ and the lens axis, and $R(\theta)$ is the reflectance function of the specimen. The latter function, for acoustic waves, can be found by solving the acoustic Fresnel equation. The reflection coefficient [15, 16] from a layer reads as

$$R(\theta) = \frac{Z_{\text{in}} - Z_{\text{liq}}}{Z_{\text{in}} + Z_{\text{liq}}}, \quad (3)$$

where $Z_{\text{liq}}$ is the impedance of plane wave in the liquid, $Z_{\text{in}}$ is the input impedance of the layer that is the impedance at the layer–liquid boundary, which is expressed by the formula:

$$Z_{\text{in}} = Z_{\text{ch}} \frac{Z_{\text{sub}} - iZ_{\text{ch}}tg\phi}{Z_{\text{ch}} - iZ_{\text{sub}}tg\phi} \quad (4)$$

with $\phi = k_Lh_L\cos\theta_L$ being the phase advance of the plane wave passing through the layer of an $h$ thickness, and $Z_{\text{sub}}$ and $Z_{\text{ch}}$ are the acoustic impedances of substrate and layer given by

$$Z_i = \rho_i V_i / \cos\theta_i, \quad (5)$$

where subscript $i = \text{liq}, \text{ch}$ or $\text{sub}$ stands for liquid, layer, or substrate, respectively. It is clear that, at normal incidence, the acoustic impedance becomes simply the product of density and velocity. Hence, the intensity reflection coefficient of a layer on a substrate is as follows:

$$R = \frac{Z_{\text{ch}}^2(Z_{\text{sub}} - Z_{\text{liq}})^2\cos^2k_{\text{ch}}h + (Z_{\text{sub}}Z_{\text{liq}} - Z_{\text{ch}}^2)^2\sin^2k_{\text{ch}}h}{Z_{\text{ch}}^2(Z_{\text{sub}} + Z_{\text{liq}})^2\cos^2k_{\text{ch}}h + (Z_{\text{sub}}Z_{\text{liq}} + Z_{\text{ch}}^2)^2\sin^2k_{\text{ch}}h}. \quad (6)$$
Note that the reflection coefficient is a complex-valued function with an amplitude and a phase and the total reflections obtained for \(|R(\theta)| = 1\). Therefore, the \(V(z)\) calculation from relation (1) can readily be carried out by just knowing the SAW velocities and material densities.

**Acoustic Velocity Determination.** The schematic representation of \(V(z)\) curves, given by Eq. (1), is shown in Fig. 2, \(a\); it consists of many peaks and valleys due to constructive and destructive interference between different propagating modes, with a main peak at the focal distance \((z = 0)\) representing the lens response. However, successive peaks decay exponentially when \(z\) increases, because of the influence of the acoustic lens signal, \(V_{\text{lens}}\) (Fig. 2, \(b\)). Thus, the real signal of the specimen, \(V_s(z)\), would be

\[
V_s(z) = V(z) - V_{\text{lens}}(z).
\]  

(7)

Thus, the obtained signal (Fig. 2, \(c\)) is a periodic curve characterized by a spatial period \(\Delta z\). Hence, its treatment can be carried out via fast Fourier transform (FFT), which exhibits a large spectrum consisting of one or several peaks (Fig. 2, \(d\)).

The dominant mode (usually Rayleigh one) appears as a very sharp and pronounced peak, from which the Rayleigh velocity can be determined [8] according to the relation:

\[
V_R = \frac{V_{\text{liq}}}{\sqrt{1 - \left(\frac{V_{\text{liq}}}{2f\Delta z}\right)^2}},
\]  

(8)

where \(V_{\text{liq}}\) is the sound velocity in the coupling liquid and \(f\) the operating frequency.

**Elastic Constants Determination.** It is well known that the Rayleigh velocity is generally determined experimentally from SAM, satis-
fying the standard equation. In order to determine the elastic constants $E$ and $G$, Viktorov’s formula was used [10]:

$$V_R = V_T \frac{0.718 - (V_T/V_L)^2}{0.75 - (V_T/V_L)^2}.$$  

Elastic constants can be expressed in term of density $\rho$ and velocities of the longitudinal $V_L$ and transverse $V_T$ modes of acoustic waves [11].

$$E_1 = \frac{\rho V_T^2 [3V_L^2 - 4V_T^2]}{V_L^2 - V_T^2}, \quad (10)$$

$$G_1 = \rho V_T^2. \quad (11)$$

On the other hand, another approach has been proposed [17] to find the relationships between the velocities of the different modes (Rayleigh, longitudinal, and transverse ones) of acoustic waves in order to determine the Young’s module $E$ and the shear modulus $G$ by an expression that contains only one of these terms:

$$E_2 = 2.99 \rho V_R^2, \quad (12)$$

$$E_2 = 0.757 \rho V_L^2, \quad (13)$$

$$E_2 = 2.586 \rho V_T^2, \quad (14)$$

$$G_2 = 1.156 \rho V_R^2, \quad (15)$$

$$G_2 = 0.293 \rho V_L^2. \quad (16)$$

The application of these equations removes several limitations related to SAM operational conditions.

**Materials and Simulation Conditions.** It is important to note that the determination of Rayleigh velocities of several deposited liquid metals is impossible in using the SAM technique. For this reason, the simulation of deposited metals was taken in a bulk state for determining these Rayleigh velocities, and comparison between obtained results and experimental sound velocities of several deposited liquid metals was made to enrich this study.

The calculations were approved out in case of a reflexion scanning acoustic microscope; Rayleigh mode dominate and appears under normal operating conditions (half-opening angle of lens $50^\circ$, working frequency is $142$ MHz and water as a coupling liquid whose wave velocity $V_{\text{liq}} = 1500$ m/s and density $\rho = 1000$ kg/m$^3$) or with annular lenses.

The final step consists in determination of the Rayleigh velocities from reflection coefficient and the acoustic signature; for example simulation, it will be taken two metals tin (Sn) and silicon (Si).

**Reflection Coefficient.** The reflection function, $R(\theta)$, was first calculated for two deposited metals (Sn and Si) to show their effects in the experimental calculation of Rayleigh velocity. The curves obtained are
shown in Fig. 3. For a better representation of the curve and since \( R(\theta) \) is a complex function, we have separated the amplitude curves (Fig. 3, \( a \)) from those of the phase (Fig. 3, \( b \)). Then, for the deposited metals mentioned above, the real parts and the imaginary parts were superposed as a function of the angles of incidence \( \theta_i \).

Following Fig. 3, \( a \), representing the amplitude of \( R(\theta) \) as a function of the angle of incidence \( \theta_i \), we can clearly observe a first amplitude fluctuation when the angle of incidence reaches the values of the critical longitudinal angles \( \theta_L \). A change from \( \theta_L \) to higher values. Then, a second fluctuation when the angle of incidence reaches the values of the critical transverse angles, \( \theta_T \). Between \( \theta_L \) and \( \theta_T \), amplitude of \( R(\theta) \) remains constant. Finally, beyond \( \theta_T \), the amplitude of \( R(\theta) \) increases to reach the unit corresponding to total reflection.

Following Fig. 3, \( b \), representing the phase of \( R(\theta) \) as a function of \( \theta_i \), it can be easily noticed that almost a \( 2\pi \) transition is obtained for Si. This transition occurs at the critical angle, \( \theta_R \), which corresponds to the Rayleigh mode, which is the most important in the current simulation conditions. Thus, the Rayleigh mode dominates all other modes leading to the fact that the longitudinal critical angle, \( \theta_L \), is not very noticeable.

It can also be seen that the amplitude of the transition in the Rayleigh mode phase becomes lower than the usual \( 2\pi \) value for Sn. While the position, i.e. the value of \( \theta_R \) moves to lower values (similar behaviour to that observed with \( \theta_L \) in Fig. 3, \( a \)). In addition, it is clear that all modes are generated with angles less than \( 20^\circ \). These critical angles strongly depend on the simulation conditions, in particular the coupling liquid densities.

**Acoustic Signature.** The acoustic signature can be calculated from the spectral angular model. The curves obtained for the two deposited metals (Sn and Si), are shown in Fig. 4.

It is clear that the two curves of \( V(z) \) exhibit an oscillatory behaviour, with a spatial period \( \Delta z \), due to constructive and destructive
interferences between the propagation modes. It should be noted that the two curves are distinct in amplitude as well as in the periods, Δz. In amplitudes, the curves attenuate faster for Si corresponding to a large period. Such behaviour is the result of previously observed changes in the module and phase curves of the reflectance functions.

The FFT spectral analysis of these periodic curves V(z) is shown in Fig. 2, b. These spectra are characterized by a principal peak representing the most dominant mode, which is that of Rayleigh, under the present conditions. However, the efficiency of this mode represented by its height is more important for higher VR. Moreover, a small shift is observed for the main ray highlighting the spatial differences Δz obtained in the curves V(z).

Several deposited metals parameters used in this investigation are listed in Table 1; sound velocities at melting temperatures are tabulated by Blairs [18], surface tension values are proposed by Keene [19], liquid densities are taken by Crawley [20] and by Baykara et al. [21]. While the elastic constants and solid densities are obtained from Briggs [10], Rayleigh velocity determined by SAM and Rayleigh velocity calculated from one-parameter approach are given in Table 1.

**Results and Quantification**

The objective of this quantification is to find correlations applicable for the estimation of surface tension and the work of adhesion for different liquid metals in contact with ceramic as function of acoustic velocities and elastic constants of these metals.
Table 1. Experimental sound velocities, $c$, surface tensions, $\sigma_m$, densities, $\rho_{lm}$, of different liquid metals at the melting temperature, elastic moduli, densities, $\rho_{sm}$, and calculated Rayleigh velocities, $V_R$, of these metals at solid state.

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Estimation of $c$ and $\rho_{lm}$ in Terms of $V_i$ and $\rho_{sm}$ for Different Solid Metals

In this article, analytical study has been proposed to express the relation between experimental sound velocities of liquid metals at the melting temperature and determinate acoustic velocities of these metals at solid state by SAM program.

It is noted that Rayleigh velocities of bulk metals determined by SAM have almost the same values that these calculated by one parameter approach using Eq. (11).

The variation of $V_R$ values as function of $c$ was made; it shows a linear increase of $V_R$ with $c$ increasing. Simple fitting was made and resulted in a well-defined linear correlation between the quantities, as can be seen in Fig. 5.

The quantified correlation between $V_R$ and $c$ can be written as

$$V_R = 0.674c. \quad (17)$$

Longitudinal and transverse velocities follow similar behaviours that take the following form:

$$V_L = 1.342c, \quad (18)$$

$$V_T = 0.724c. \quad (19)$$

Relation between acoustic velocities of solid metals and experimental sound velocities of liquid metals can be generalized with following analytical form:

$$V_i = A_i c \quad (20)$$

where $A_i$ is a characteristic constant for velocities; the subscripts $i = L, T, R$ represent the propagating longitudinal, transverse, and Rayleigh waves modes.

---

Fig. 5. Correlation between experimental sound velocities $c$ for liquid metals and calculated Rayleigh velocities $V_R$ of these metals in solid state

Fig. 6. Correlation between densities for liquid ($\rho_{lm}$) and solid ($\rho_{sm}$) metals
One can see also a clear tendency between the liquid-metals’ densities, $\rho_{lm}$, with that of these metals at solid state, $\rho_{sm}$, as can be seen in Fig. 6. The relationship that expresses this tendency can take the following form:

$$\rho_{sm} = 1.088 \rho_{lm}. \quad (21)$$

The importance of Eqs. (20) and (21) lies in the prediction of acoustic parameters from liquid to solid states of metals and *vice versa*.

**Determination of $\sigma_m$ in Terms of the Acoustic Velocities for Different Metals**

Many statistical theories established to associate the surface tension and sound velocities [22–25]. In this context, Auerbach proposed semi-empirical relation to express the sound velocity of such liquid metal at the melting temperature in term of $\sigma_m$ and $\rho_{lm}$ [26]:

$$c = \frac{\sigma_m}{6.33 \cdot 10^{-10} \rho_{lm}}. \quad (22)$$

According to Mayer [27], the previous equation can be written as:

$$c = A \left( \frac{\rho_{lm}}{\rho_{sm}} \right)^{1/2} (\sigma_m / \rho_{lm})^{1/2}, \quad (23)$$

where $A$ is a constant, $V_m$ is the molar volume, and $\gamma$ is the ratio of the isobaric, $C_p$, and isochoric, $C_v$, heat capacities. The plot of $\log(c)$ versus $\log(\sigma_m / \rho_{lm})$ may be linear with a slope equal to 0.67 for Auerbach relation and equal to 0.50 for Mayer relation. This point was study by Blairs [18]; slope equal to 0.552 is found.

In this context, to analyse the functional dependence of $\sigma_m$ and $V_R$ of solid metals, a linear correlation between the behaviours of quantities is found, where the deduced $V_R$ values increase with increasing of the quantity $(\sigma_m / \rho_{sm})$, as can be seen in Fig. 7.

To quantify the relation between $V_R$ and $(\sigma_m / \rho_{sm})$, a logarithmic plot was made in the present work; a linear correlation between the quantities is defined by:

$$\log(V_R) = 0.49 \log (\sigma_m / \rho_{sm}) + 8.53. \quad (24)$$

It would be noted that similar behaviours were deduced for longitudinal and transverse velocities; this is evident in the following relations:

$$\log(V_L) = 0.49 \log (\sigma_m / \rho_{sm}) + 9.22, \quad (25)$$

$$\log(V_T) = 0.49 \log (\sigma_m / \rho_{sm}) + 8.60. \quad (26)$$

The above relations of acoustic velocities take the following general form:

$$\log(V_i) = 0.49 \log (\sigma_m / \rho_{sm}) + B_i, \quad (27)$$

where $B_i$ is characteristic constants for velocities.

Equation (27) shows that the slope of the plot of $\log(V)$ versus $\log(\sigma_m / \rho_{sm})$ is closer to the Mayer proposition than that of the Auerbach
Z. Hadef, A. Doghmane, K. Kamli, and Z. Hadjoub

relation. It is finds application for the estimation of unknown surface tension of metallic liquids using available acoustic velocities and density values in solid state.

**Determination of $\sigma_m$ in Terms of Elastic Constants for Various Metals**

A close comparative of Eqs. (12), (17), and (23) shows a linear dependence between $E$ and $\sigma_m$, as can be seen in Fig. 8, where the results’ analysis shows increase comportment for surface tension with Young’s modulus increasing according to linear relationship; similar behaviour was also obtained for shear modulus.

To quantify the relation between elastic moduli and $\sigma_m$, a simple plot was made; a linear correlation is defined that it can be written as

$$E = 0.083 \sigma_m,$$  
$$G = 0.032 \sigma_m.$$  

A close comparative analysis of the above equations derived expressions shows that they can be taking the following form:

$$M = A_m \sigma_m,$$  

where $A_m$ is characteristic constant for elastic moduli.

An important point that can be interpreted from Eq. (25) is the possibility of determining the unknown surface tension for liquid metals as function of elastic moduli.

![Fig. 7. Correlation between deduced Rayleigh velocities ($V_R$) and ratio of surface tension to density in solid state ($\sigma_m/\rho_m$).](image1)

![Fig. 8. Correlation between surface tension $\sigma_m$ of different liquid metals and Young’s modulus $E$ of these metals.](image2)
Relationship between $W_{ad}$ in Metals/Ceramic Systems and Acoustic Velocities

The principle of the present approach is to find a relation between the acoustic velocities of different metals and the work of adhesion in this several metals on a given ceramic. Experimental results of the work of adhesion and the contact angle for various metal-ceramic systems are summarized in Table 2. It should be taken that the criterion of metal–ceramic systems selected in Tables 2 and 3 is the existence in the literature.

From the plotting of $W_{ad}$ values given in Table 2 against the Rayleigh velocities of the various metals, a linear correlation between these quantities is observed, where $W_{ad}$ increase with $V_R$ increasing as can be seen in Fig. 9 for different metal–aluminium nitride systems.

It is reasonably to express the quantified correlation between $W_{ad}$ and $V_R$ as can be written as

$$W_{ad} = 0.434 V_R.$$  \hfill (31)

Table 2. Experimental values of the work of adhesion $W_{ad}$ for different metal/ceramic systems

<table>
<thead>
<tr>
<th>Ceramics</th>
<th>Metal</th>
<th>Atmosphere</th>
<th>$W_{ad}$ (mJ/m²)</th>
<th>Refs.</th>
<th>Ceramics</th>
<th>Metal</th>
<th>Atmosphere</th>
<th>$W_{ad}$ (mJ/m²)</th>
<th>Refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlN</td>
<td>Au</td>
<td>Vacuum</td>
<td>550</td>
<td>[28]</td>
<td>BeO</td>
<td>Cu</td>
<td>Ar</td>
<td>600</td>
<td>[31]</td>
</tr>
<tr>
<td></td>
<td>Co</td>
<td>Vacuum</td>
<td>1270</td>
<td>[28]</td>
<td>Fe</td>
<td>He</td>
<td></td>
<td>717</td>
<td>[32]</td>
</tr>
<tr>
<td></td>
<td>Cu</td>
<td>Vacuum</td>
<td>1060</td>
<td>[29]</td>
<td>Ni</td>
<td>Vacuum</td>
<td></td>
<td>680</td>
<td>[32]</td>
</tr>
<tr>
<td></td>
<td>Fe</td>
<td>Vacuum</td>
<td>1320</td>
<td>[28]</td>
<td>Pb</td>
<td>Vacuum</td>
<td></td>
<td>130</td>
<td>[33]</td>
</tr>
<tr>
<td></td>
<td>Ga</td>
<td>Vacuum</td>
<td>750</td>
<td>[28]</td>
<td>BN</td>
<td>Au</td>
<td>Vacuum</td>
<td>205</td>
<td>[34]</td>
</tr>
<tr>
<td></td>
<td>Ge</td>
<td>Vacuum</td>
<td>911</td>
<td>[28]</td>
<td>Cu</td>
<td>Vacuum</td>
<td></td>
<td>345</td>
<td>[34]</td>
</tr>
<tr>
<td></td>
<td>In</td>
<td>Vacuum</td>
<td>448</td>
<td>[28]</td>
<td>Si</td>
<td>Vacuum</td>
<td></td>
<td>664</td>
<td>[34]</td>
</tr>
<tr>
<td></td>
<td>Ni</td>
<td>Vacuum</td>
<td>1305</td>
<td>[28]</td>
<td>Sn</td>
<td>Vacuum</td>
<td></td>
<td>128</td>
<td>[34]</td>
</tr>
<tr>
<td></td>
<td>Pb</td>
<td>Vacuum</td>
<td>203</td>
<td>[28]</td>
<td>MgO</td>
<td>Ag</td>
<td>Ar</td>
<td>421</td>
<td>[34]</td>
</tr>
<tr>
<td></td>
<td>Pd</td>
<td>Vacuum</td>
<td>858</td>
<td>[28]</td>
<td>Fe</td>
<td>Vacuum</td>
<td></td>
<td>820</td>
<td>[35]</td>
</tr>
<tr>
<td></td>
<td>Sn</td>
<td>Vacuum</td>
<td>461</td>
<td>[29]</td>
<td>Ni</td>
<td>He</td>
<td></td>
<td>585</td>
<td>[37]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sn</td>
<td>Vacuum</td>
<td></td>
<td>278</td>
<td>[36]</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>Al</td>
<td>Vacuum</td>
<td>948</td>
<td>[30]</td>
<td>NiO</td>
<td>Ag</td>
<td>Ar</td>
<td>1267</td>
<td>[35]</td>
</tr>
<tr>
<td></td>
<td>In</td>
<td>Vacuum</td>
<td>335</td>
<td>[30]</td>
<td>Cu</td>
<td>Vacuum</td>
<td></td>
<td>390</td>
<td>[34]</td>
</tr>
<tr>
<td></td>
<td>Ni</td>
<td>Vacuum</td>
<td>1191</td>
<td>[30]</td>
<td>Si</td>
<td>Ar</td>
<td></td>
<td>708</td>
<td>[38]</td>
</tr>
<tr>
<td></td>
<td>Pb</td>
<td>Vacuum</td>
<td>218</td>
<td>[30]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pd</td>
<td>Vacuum</td>
<td>704</td>
<td>[30]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sn</td>
<td>Vacuum</td>
<td>305</td>
<td>[30]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Similar behaviours of other acoustic velocities would also be deduced; this is evident in the form:

\[ W_{ad} = 0.864V_L, \]  
\[ W_{ad} = 0.467V_T. \]

The above relations take the following general form:

\[ W_{ad} = C_i V, \]

where \( C_i \) is slope parameter, which gives the interfacial adhesion as function of acoustic velocities.

The points presented in Fig. 9 yields a slope parameter for Al–N. These results, together with the \( C_R \) values obtained for other metals–ceramics systems, are given in Table 3.

Moreover, it should be noted that Eq. (34) and \( C_R \) values presented in Table 3 determine directly the work of adhesion of different metals–ceramics systems depending on the acoustic proprieties of these metals.

### Table 3. Slope parameter of Rayleigh velocities, \( C_R \), and the coefficient of the linear regression, \( R \), determined for various ceramic

<table>
<thead>
<tr>
<th>Ceramic</th>
<th>( C_R )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlN</td>
<td>0.434</td>
<td>0.972</td>
</tr>
<tr>
<td>Al(_2)O₃</td>
<td>0.363</td>
<td>0.931</td>
</tr>
<tr>
<td>BeO</td>
<td>0.247</td>
<td>0.986</td>
</tr>
<tr>
<td>BN</td>
<td>0.137</td>
<td>0.972</td>
</tr>
<tr>
<td>MgO</td>
<td>0.236</td>
<td>0.934</td>
</tr>
<tr>
<td>NiO</td>
<td>0.859</td>
<td>0.960</td>
</tr>
<tr>
<td>SiO₂</td>
<td>0.151</td>
<td>0.976</td>
</tr>
</tbody>
</table>

Relation between \( W_{ad} \) in Metals/Ceramics Systems and Elastic Constants

To enrich this estimation, it would be useful to quantify the influence of elastic constants on the work of adhesion. The plot of \( \log(W_{ad}) \) against \( \log\left(\frac{E}{\rho_{sm}}\right) \) is evaluated. Some typical results are summarized in Fig. 10; it is clear that the general tendency is for an increase in \( W_{ad} \) as \( \left(\frac{E}{\rho_{lm}}\right) \) increases.

Using a simple logarithmic plot, it is possible to find linear dependence between logarithms of \( \left(\frac{E}{\rho_{sm}}\right) \) and \( W_{ad} \) as follows:

\[ \log(W_{ad}) = 0.504\log\left(\frac{E}{\rho_{sm}}\right) - 0.648, \]  
\[ \log(W_{ad}) = 0.502\log\left(G/\rho_{sm}\right) + 2.138. \]

The above elastic moduli (Young’s and shear ones) follow similar behaviours, which take the following form:

\[ \log(W_{ad}) = 0.502\log\left(M/\rho_{sm}\right) + C_m, \]

where \( C_m \) is characteristic coefficient for the elastic moduli depending on the nature of ceramic.

This behaviour has been remarked for all ceramics in contact with several liquid metals studied. In this context, a new semi-empirical relation can be proposed to express the work of adhesion in term of \( E \) and \( \rho_{sm} \) as can be seen in Fig. 11.

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Linear correlation between these quantities is defined; it can be written as:

\[ W_{ad} = 8.08 \left( \frac{E}{\rho_{sm}} \right)^{1/2} - 41.5, \]  
\[ W_{ad} = 11.75 \left( \frac{E}{\rho_{sm}} \right)^{1/2} + 52.9. \]

A close comparative analysis of above-mentioned equations derived from expressions shows that they could be taking the following form:

\[ W_{ad} = \xi \left( \frac{M}{\rho_{sm}} \right)^{1/2} + D_c, \]  

where \( \xi \) represents the slope parameter of dependence on these quantities, and \( D_c \) is characteristic coefficient for the elastic moduli depending on the nature of ceramic.

An important point that can be taken from Eq. (40) is the possibility of prediction of work of adhesion in liquid metals/ceramic systems from elastic moduli and densities of liquid metals.
Conclusion

In this work, surface tension and work of adhesion of different liquid metals on a given ceramic are investigated. Acoustic parameters (namely, longitudinal, transverse, and Rayleigh velocities) and elastic constants (namely, Young’s and shear moduli) are calculated for all solid metals of the system at issue. It is shown that these parameters change with increasing surface tension as well as with increasing work of adhesion. Applying different complex-quantitative methods [39–43] for the analysis and quantification, we found new linear semi-empirical formulas, which express the variations of velocities and elastic constants. The importance of this estimation consists in the prediction of acoustic parameters for any surface tension and work of adhesion in metals/ceramic interfaces and vice versa.

REFERENCES

Tension, Adhesion in Liquid Metals/Ceramic Systems, and Acoustic Parameters


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The expressions have the following form: 

\[ \log(V_i) = 0.49 \log(\sigma_m/\rho_{sm}) + B_i, \quad M = A_m \sigma_m, \]

\[ W_{ad} = C_i V_i \quad \text{and} \quad W_{ad} = \xi (M/\rho_{sm})^{1/2} + D_i, \]

where \( A_i, B_i, C_i, A_m, \xi \) and \( D_m \) are characteristic values of velocities and moduli of elasticity, while indices \( m \) refers to moduli of elasticity (Young or shear), and indices \( i = L, T, R \) — to propagating longitudinal, transverse or Rayleigh waves. The importance of the obtained formulas lies in the possibility of predicting the surface tension and adhesion work at the metal/ ceramic interface depending on the elastic and acoustic characteristics.

**Key words:** surface tension, adhesion work, acoustic velocities, elastic constants, ceramic, liquid metals, interface.

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**CORRELATION BETWEEN SURFACE TENSION, ADHESION WORK IN SYSTEMS LIQUID METALS/ CERAMICS AND ACOUSTIC PARAMETERS**

In the article, we investigate and demonstrate for the first time the correlation between acoustic velocities \( V \), elastic moduli \( M \), densities \( \rho \) and surface tension \( \sigma_m \) and work of adhesion \( W_{ad} \) of various liquid metals on a fixed ceramic. A numerical program for scanning acoustic microscopy (SAM) is used in conditions favorable for generating acoustic waves. It was found that for specific systems, all studied acoustic parameters show a good dependence on the surface tension and the adhesion work. Analysis and quantitative determination of the results led to the establishment of semiparametric formulas. The obtained expressions have the following form:

\[ \log(V_i) = 0.49 \log(\sigma_m/\rho_{sm}) + B_i, \quad M = A_m \sigma_m, \]

\[ W_{ad} = C_i V_i \quad \text{and} \quad W_{ad} = \xi (M/\rho_{sm})^{1/2} + D_i, \]

where \( A_i, B_i, C_i, A_m, \xi \) and \( D_m \) are characteristic values of velocities and moduli of elasticity, while indices \( m \) refers to moduli of elasticity (Young or shear), and indices \( i = L, T, R \) — to propagating longitudinal, transverse or Rayleigh waves. The importance of the obtained formulas lies in the possibility of predicting the surface tension and adhesion work at the metal/ ceramic interface depending on the elastic and acoustic characteristics.

**Key words:** surface tension, adhesion work, acoustic velocities, elastic constants, ceramic, liquid metals, interface.